Real-World Applications of Small-World Networks; intermediate connection topology as a model for social, biological, and technological networks

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Abstract

The predictions made by Watts and Strogatz in their foundational text were prescient: small-world networks are nearly ubiquitous. This essay considers the veracity of these networks in modelling real-world complex systems. With brief analysis of the referenced material, this essay will conclude that SWNs are observed and accurately implemented across multiple domains.

Keywords: Small-world networks, Complex systems, Small-world phenomenon, Network models

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In their formative work, Watts and Strogatz (1998) present a class of networks which possess a unique combination of characteristics. Small-world networks ("SWMs") are highly clustered yet demonstrate small characteristic path lengths ("CPL"); in some sense, these networks are an amalgam of the contrary extremes of connection topology: regular lattices and random graphs (Watts & Strogatz, 1998). As such, SWNs may be considered an intermediate connection topology which are witnessed at some point on the spectrum between entirely rigid and stochastic structures. Figure 1 illustrates (see Appendix A) that, sufficient some rewiring probability $0<p<1$, SWNs exhibit features of regular as well as random networks:

- like regular lattices, they are highly clustered.
- like random networks, they have short CPL.

It is clear from the figure that SWNs contain several "short cuts" between vertices. Watts and Strogatz (1998) note that, for small *p*, the marginal short cut has a highly nonlinear effect on CPL whilst leaving the degree of clustering relatively unchanged. Consequently, SWNs simultaneously exhibit regional specialisation and enhanced signal-propagation speed—a desirable synthesis for complex networks (e.g. network of interconnected specialised computers sharing workload). Owing to their dualism, SWNs provide a practical paradigm for analyzing observable networks. Furthermore, the principles of SWNs are useful in developing efficient systems given their unique ability to process and transfer information; to this end, Watts and Strogatz (1998) expect the small-world phenomenon to be widespread in social, biological, and man-made systems. The purpose of this essay is to consider the veracity of SWNs in the context of these three diverse domains.

In their study on alcohol epidemiology, Braun, Wilson, Pelesko, Buchanan, and Gleeson (2006) utilise SWNs in an attempt to determine the smallest percentage of alcohol dependents needed to be treated to reduce the prevalence of alcohol abuse in structured communities, such as college campuses. The researchers apply a mathematical model to conceptualise the mechanism by which alcoholism is spread; namely, the drinking roles and norms conveyed by neighbours in the network. Braun et al. (2006) employ SWNs to model these structured communities due to the capacity of such networks to incorporate different forms of connectivity membership—the SWN is able to model both the acquaintance between, and relative whereabouts of, individuals in a community. Additionally, local clustering enables the researchers to accurately model subgroups within a larger ecology; which is practical given that within communities, such as college campuses, subgroups (e.g. social cliques) will clearly influence the rate at which drinking norms are propagated. Braun et al. (2006) modify the SWN to include a function of resilience to alcoholism and subsequently alter structural parameters of the network to find that the dispersion of alcohol-related behaviour is controlled by these characteristics in the community; for example, a low average value of resilience results in a population dominated by alcohol dependency (vice versa)—put simply, the researchers find that shorter CPL and less clustering in a community network promotes the spread of alcoholism (Braun, Wilson , Pelesko, Buchanan, & Gleeson , 2006). As a result, any factors which influence CPL and the degree of clustering in the network are expected to influence the prevalence of dependency (Braun, Wilson , Pelesko, Buchanan, & Gleeson , 2006). The salient result of this research is that policy decisions concerning treatment and prevention of alcohol-related behaviours in communities will demonstrably influence outcomes.

In Liu, Li, Qin, Liu, Wang, and Wang's (2015) study of the susceptible-infected-susceptible (SIS) model of epidemic spreading, the authors interestingly deviate from the established epidemiological applications of SWNs seen in the literature; rather than focusing their analysis on a single SWN in isolation, Liu et al. examine a system comprising two interconnected SWNs. Liu et al. recognise the extent to which epidemic dynamics in complex networks—predominantly single networks (e.g. Watts and Strogatz)—have already been studied. The authors put forward an important shortcoming of these studies; namely, epidemics are not only realised in single-networks (Liu, et al., 2015). Epidemics may spread across several communities or between species (*cross-species transmission*); for example, zoonotic diseases, such as Ebola and Zika virus, are spread from animal interaction networks to the human interaction network. To give an insight into the importance of incorporating interconnected networks into epidemiological analysis, Liu et al. note that the epidemic threshold in two interconnected random networks has been shown to be lower than in the separate networks. Analogously, the main finding of Liu et al.'s (2015) paper is that the epidemic threshold in interconnected SWNs decreases with an increase in p. Additionally, the authors ascertain that when the infection rate is low, p has a pronounced positive effect on the global steady-state infection density whereas, when the infection rate is high, infection density is insensitive to changes in p (Liu, et al., 2015). This result is intuitive: when the probability of infection spreading from one node to another is small, the relative importance of short cuts in the networks increases as CPL has greater influence on global infection density. Evidently, a better understanding of epidemiology in the context of interconnected SWNs is desirous. Not all epidemiological phenomena can be captured by a single network (Liu, et al., 2015). Through completing nascent analysis of epidemics in interconnected networks, Liu et al. invite further research; they deem immunization and vaccination strategies as the exigent future work.

In applying a SWN model to the Boston underground transportation system (“MBTA”, see Figure 2, Appendix A), Latora and Marchiori (2002) highlight some important issues with passing from abstract classes of networks (e.g. social networks) to real complex systems, such as transportation networks; specifically, Latora and Marchiori (2002) note that:

- SWNs are only applicable in certain cases; and
- SWNs only operate in *topological abstraction.*

Clearly, these issues present a challenge to arbitrary attempts to utilise SWNs in analysis of real-world systems; pertaining to the first issue, in real complex systems the degree of clustering and CPL are ill-defined—constraining the potential for any meaningful analysis (Latora & Marchiori, 2002). That analysis can only be achieved through the abstraction of a network’s characteristics (i.e. SWNs convey the presence/absence of a link but contain no additional information about the link itself, such as length) severely limits the usefulness of the Watts-Strogatz model in the analysis of complex real-world networks (Latora & Marchiori, 2002). To circumvent the aforementioned issues, Latora and Machiori (2002) develop a SWN representation of the MBTA—for which clustering and CPL are well-defined— and implement a measure of efficiency into their model. Based on this measure, the MBTA is found to be a very efficient transport system on a global scale (only 37% less efficient than an ideal station-to-station subway) but not at a local level (Latora & Marchiori, 2002). The network is not fault tolerant, the loss of one node (station) will drastically diminish the efficiency between an affected station and its neighbours; why then does the SWN construction persist? Considering the cost-effectiveness of the SWN, Latora and Marchiori (2002) determine that temporary problems in individual stations can be solved in economic ways by transporting users by other means (e.g. providing a replacement bus to the next station). Evidently, the MBTA is not a closed system; it is a subgraph of a wider transport network. Therefore, global efficiency in transport networks can be achieved at relatively low cost using the SWN as a construction principle (Latora & Marchiori, 2002). This paper illustrates the value of SWNs in developing and modelling real-world complex systems, such as transportation networks. Saliently, Latora and Marchiori (2002) recognise that we often only have a partial view of complex systems and demonstrate that modifying our abstract representations of these systems can provide greater insight.

Considering the above discussion, it seems reasonable to conclude that many of the predictions made by Watts and Strogatz (1998) were largely accurate. Subsequent inquiries into the nature of SWNs and their applications appear to have done nothing but validate the foundational text. In discussing the literature, it is shown here that Watts and Strogatz’s (1998) expectations for SWNs to be widespread in social, biological, and man-made systems are confirmed. Furthermore, it can be reasonably inferred from the referenced material that intermediate connection topology is accurate in modelling real-world complex systems, ranging from local communities to transportation networks—evidently, there are myriad real-world applications of SWNs. To this end, the literature would suggest that these networks are so prevalent in nature because they possess many desirable characteristics: adaptability, efficiency, and robustness. Consequently, SWNs would seem to be extremely important to a diverse range of fields, including:

- biology
- computer science
- economics
- engineering
- epidemiology
- mathematics
- sociology

The referenced material makes apparent that any application of SWNs should take into account the limitations of the model, as made clear by Liu et al. (2015). Similarly, analysis of SWNs found in nature should consider that these networks are also likely to be subgraphs within larger systems, as highlighted by Latora and Marchiori (2002); in recognising this, agents can drastically improve the effectiveness of the analysis/application of SWNs in the real-world. Improvements in understanding can always be attained; for the referenced material, it would be suggested that greater insight could be achieved by authors expanding the scope of their studies; for example, Braun et al. (2006) may study entire cities or regions, Liu et al. (2015) may increase the number of interconnected networks in their modelling, and Latora and Marchiori (2002) may analyse other subway-systems or even other types of transport networks. That said, despite these suggestions, the referenced material provided valuable insight into real-world applications of small-world networks.

References

Braun, R., Wilson , R., Pelesko, J., Buchanan, J., & Gleeson , J. (2006). Applications of small-world network theory in alcohol epidemiology . *Journal of Studies on Alcohol*, 591-9.

Latora, V., & Marchiori, M. (2002). Is the Boston subway a small-world network? *Physica A: Statistical Mechanics and its Applications* , 109-113.

Liu, M., Li, D., Qin, P., Liu, C., Wang, H., & Wang, F. (2015). Epidemics in Interconnected Small-World Networks. *PLOS ONE*.

Watts, D. J., & Strogatz, S. H. (1998). Collective dynamics of 'small-world' networks . *Nature*, 440-442.

Appendix A

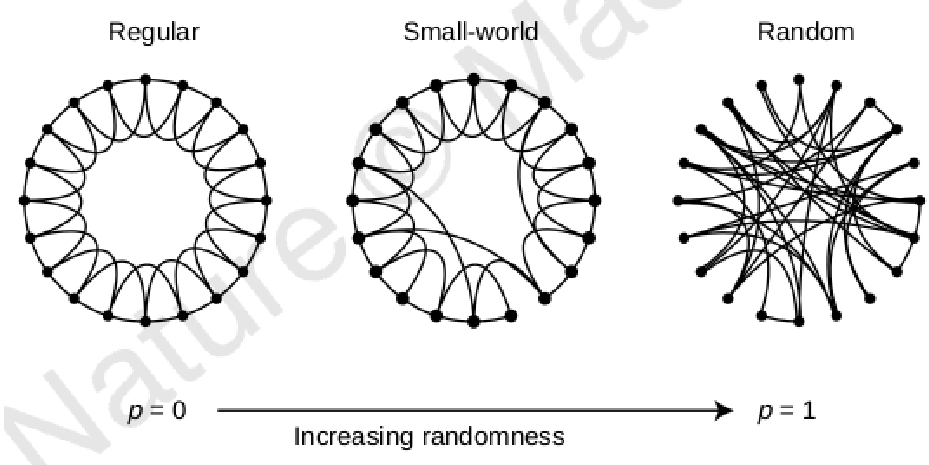

Figure 1: Random rewiring procedure between regular ring lattice and a random network (Watts & Strogatz, 1998)

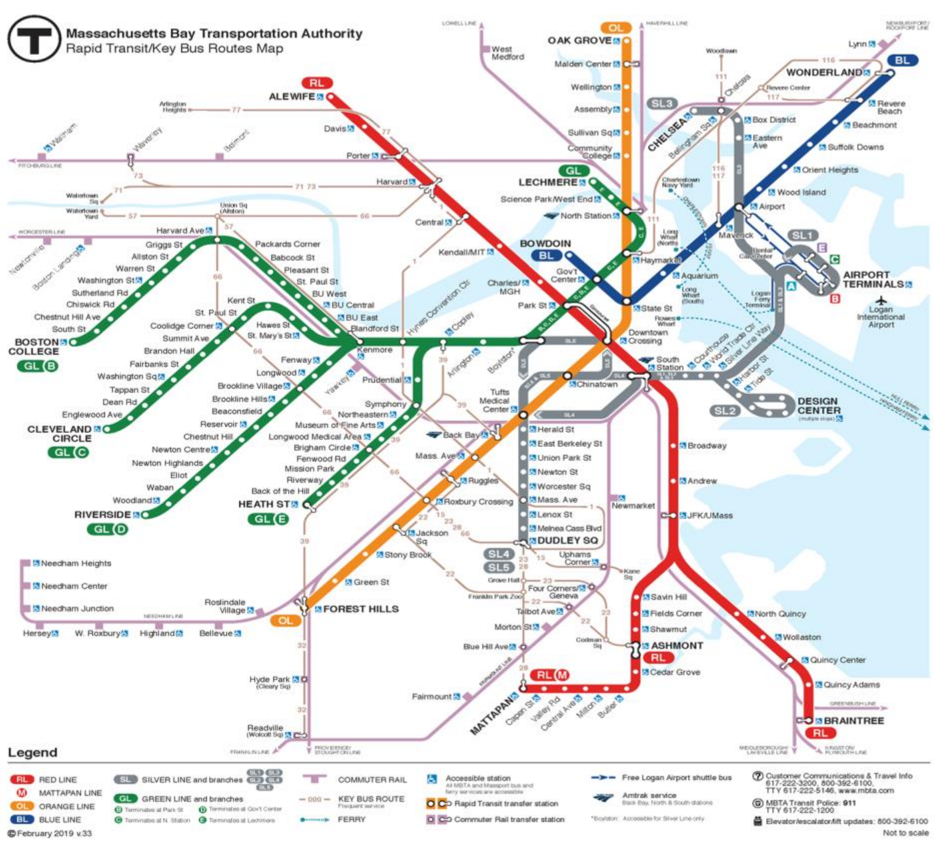

Figure 2: MBTA network (www.mbta.com/schedules/subway)